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TRANSVERSAL HYPERSURFACES OF ALMOST HYPERBOLIC CONTACT MANIFOLDS ENDOWED WITH SEMI SYMMETRIC NON-METRIC CONNECTION

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Abstract. In this study, transversal hypersurfaces of almost hyperbolic contact manifolds endowed with a semi symmetric non metric connection has been introduced. Some results regarding transversal hypersurfaces of almost hyperbolic contact manifold endowed with a semi symmetric non metric connection have been obtained in this context. Transversal hypersurfaces of cosymplectic hyperbolic manifold and trans hyberbolic Sasakian manifold endowed with a semi symmetric non metric connection are also studies.

Keywords: almost contact metric manifold, trans hyperbolic contact manifold hypersurface, semi symmetric non metric connection, transversal hypersurfaces of a cosymplectic hyperbolic manifold, transversal hypersurface of a trans hyperbolic Sasakian manifold.

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1. Introduction

Let ∇ be a linear connection in an n-dimensional differentiable manifold M. The torsion tensor T and the curvature tensor R of ∇ are given respectively by

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,\ Y],$$
$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z.$$

The connection ∇ is symmetric if its torsion tensor T vanishes, otherwise it is non-symmetric. The connection ∇ is a metric connection if there is a Riemannian metric g in M such that $\nabla g = 0$, otherwise it is non-metric. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection.

In ([9], [23]) A. Friedmann and J.A. Schouten introduced the idea of a semi-symmetric linear connection in a differentiable manifold. A linear connection is said to be a semi-symmetric connection if its torsion tensor T is of the form

$$T(X,Y) = u(Y)X - u(X)Y,$$

where u is a 1-form. In [28] K. Yano considered a semi-symmetric metric connection and studied some of its properties. In [1], [2], [24] and [26], some kind of semi-symmetric non-metric connections were studied. On other hand, there is almost contact metric manifold with an almost contact metric structure is very well explained by Blair [4]. In [25] S. Tanno gave a classification for connected almost contact metric manifolds whose automorphism groups have the maximum dimension. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. He showed that they can be divided into three classes: (1) Homogenous normal

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contact Riemannian manifolds with $c > 0$, (2) global Riemannian products of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature if $c = 0$ and (3) a warped product space $R \times f^{C^n}$ if $c < 0$. It is known that the manifolds of class (1) are characterized by some tensorial relations admitting a contact structure. Kenmotsu [15] characterized the differential geometric properties of the third case by tensor equation $\left(\overline{\nabla}_X \phi\right) Y = g\left(\phi X, Y\right)\xi - -\eta(Y)\phi X$. The structure so obtained is now known as Kenmotsu structure. In general, these structures are not Sasakian [13].

Oubina studied a new class of almost contact Riemannian manifold known as trans-Sasakian manifold [11] which generalizes both α-Sasakian [11] and β-Kenmotsu [11] structure.

M. D. Upadhayay studied almost contact hyperbolic $(f, g, \ \eta, \xi)$-structure [27]. Bhatt and Dubey studied on CR-submanifolds of trans hyperbolic contact manifold [3]. B. Y. Chen studied Geometry of submanifolds and its applications. Sci. Univ Tokyo. Tokyo, 1981. [6]. R. Prasad, M. M. Tripathi, J. S. Kim and J-H. Cho., studied some properties of submanifolds of almost contact manifold [18]-[22].

2. Preliminaries

Let $\overline{M}$ be an $2n + 1$ dimensional manifold with almost hyperbolic contact metric structure (ϕ, ξ, η, g), where ϕ is a tensor field of type (1, 1), ξ is a vector field, η is a 1-form and g is the semi Riemannian metric on $\overline{M}$. Then the following conditions [27] are satisfied

$$\phi^2 X = X + \eta(X)\xi, \ \ \phi\xi = 0, \ \ \eta o \phi = 0, \ \ \eta\left(\xi\right) = -1, \tag{1}$$

$$g\left(\phi X, \phi Y\right) = -g\left(X, Y\right) - \eta\left(X\right)\eta(Y), \tag{2}$$

$$g\left(\phi X, \ Y\right) = -g\left(X, \phi Y\right) \tag{3}$$

for vector fields $X, \ Y$ on $\overline{M}$. An almost hyperbolic contact metric structure (ϕ, ξ, η, g) on $\overline{M}$ is called trans hyperbolic contact [3] if and only if

$$\left(\overline{\nabla}_X \phi\right) Y = \alpha\left\{g\left(X, Y\right)\xi - \eta(Y)X\right\} + \beta\left\{g\left(\phi X, Y\right)\xi - \eta(Y)\phi X\right\}, \tag{4}$$

for all smooth vector fields $X, \ Y$ on $\overline{M}$ and $\alpha, \ \beta$ non zero constant, where $\overline{\nabla}$ is the Levi-civita connection with respect to g. From (4) it follows that

$$\overline{\nabla}_X \xi = \alpha\phi X + \beta\left\{X + \eta(X)\xi\right\}, \tag{5}$$

for all smooth vector fields $X, \ Y$ on $\overline{M}$.

On other hand, a semi symmetric non metric connection $\overline{\nabla}$on M is defined by

$$\overline{\nabla}_X Y = \overline{\nabla}^*_X Y + \eta\left(Y\right) X. \tag{6}$$

Using (1) and (6) in (4) and (5), we get respectively

$$\left(\overline{\nabla}_X \phi\right) Y = \alpha\left\{g\left(X, Y\right)\xi - \eta(Y)X\right\} + \beta\left\{g\left(\phi X, Y\right)\xi - \eta(Y)\phi X\right\} - \eta\left(Y\right)\phi X \tag{7}$$

$$\overline{\nabla}_X \xi = \alpha\phi X + \beta\left\{X + \eta(X)\xi\right\} - X. \tag{8}$$

Let M be a hypersurface of an almost hyperbolic contact manifold $\overline{M}$ equipped with an almost hyperbolic contact structure (ϕ, ξ, η). We assume that the structure vector field ξ never belongs to tangent space of the hypersurface M, such that a hypersurface is called a transversal hypersurface of an almost contact manifold. In this case the structure vector field ξ can be taken as an affine normal to the hypersurface. Vector field X on M and ξ are linearly independent, therefore we may write

$$\phi X = F\left(X\right) + \omega(X)\xi, \tag{9}$$

where F is a (1, 1) tensor field and ω is a 1-form on M.

From (9) $\phi\xi = F\xi + \omega(\xi)\xi$, or $0 = F\xi + \omega(\xi)\xi$

$$\phi 2X = F(\phi X) + \omega(\phi X)\xi, \tag{10}$$

$$X + \eta(X)\xi = F(FX + \omega(X)\xi) + \omega(FX + \omega(X)\xi)\xi,$$

$$X + \eta(X)\xi = F^2 X + (\omega o F)(X)\,\xi. \tag{11}$$

Taking into account the equation (11), we get

$$F^2 X = X, \tag{12}$$

$$F^2 = I, \tag{13}$$

$$\eta = \omega o F.$$

Thus we have

Theorem 2.1. *Each transversal hypersurface of an almost hyperbolic contact manifold endowed with a semi symmetric non metric connection admits an almost product structure and a 1-form ω.*

From (12) and (13), it follows that

$$\eta = \omega o F,$$

$$\eta(FX) = (\omega o F)\,FX,$$

$$\eta(FX) = \omega(F^2 X),$$

$$(\omega o F)\,X = \omega(X),$$

$$\omega = \eta o F. \tag{14}$$

Now, we assume that $\overline{M}$ admits an almost hyperbolic contact metric structure (ϕ, ξ, η, g). We denote by g the induced metric on M also. Then for all $X,\ Y\ \phi\ TM$, we obtain

$$g(FX, FY) = -g(X, Y) - \eta(X)\,\eta(Y) + \omega(X)\,\omega(Y). \tag{15}$$

We define a new metric G on the transversal hypersurface given by

$$G(X, Y) = g(\phi X, \phi Y) = -g(X, Y) - \eta(X)\,\eta(Y). \tag{16}$$

So,

$$G(FX, FY) = -g(FX, FY) - \eta(FX)\,\eta(FY) =$$
$$= -g(X, Y) - \eta(X)\,\eta(Y) + \omega(X)\,\omega(Y) - (\eta o F)(X)\,(\eta o F)(Y) =$$
$$= -g(X, Y) - \eta(X)\,\eta(Y) + \omega(X)\,\omega(Y) - \omega(X)\,\omega(Y) =$$
$$= -g(X, Y) - \eta(X)\,\eta(Y) = G(X, Y).$$

Then, we get

$$G(FX, FY) = G(X, Y), \tag{17}$$

where equation (12), (14), (15) and (16) are used.
Then G is semi Riemannian metric on M that is $(F,\ G)$ is an almost product semi-Riemannian structure on the transversal hypersurface M of $\overline{M}$.
Thus, we are able to state the following.

Theorem 2.2. *Each transversal hypersurface of an almost hyperbolic contact manifold endowed with a semi symmetric non metric connection admits an almost product semi-Riemannian structure.*

We now assume that M is orientable and choose a unit vector field N of $\overline{M}$, normal to M. Then Gauss and Weingarten formulae of semi symmetric non metric connection are given respectively by

$$\overline{\nabla}_X Y = \nabla_X Y + h\,(X, Y)\,N, \quad (X, Y \in TM), \tag{18}$$

$$\overline{\nabla}_X N = -HX + \lambda X, \tag{19}$$

where $\overline{\nabla}$ and ∇ are respectively the Levi-civita and induced Levi-civita connections in $\overline{M}$, M and h is the second fundamental form related to H by

$$h\,(X, Y) = g(HX, Y), \tag{20}$$

for any vector field X tangent to M, defining

$$\phi X = fX + u\,(X)\,N, \tag{21}$$

$$\phi N = -U, \tag{22}$$

$$\xi = V + \lambda\ N, \tag{23}$$

$$\eta\,(X) = v(X),$$

$$\lambda = \eta\,(N) = g(\xi,\ N), \tag{24}$$

for $X \in TM$ we get an induced hyperbolic $(f, g,\ u, v, \lambda)$-structure on the transversal hypersurface such that

$$f^2 = I + u \otimes U + v \otimes V, \tag{25}$$

$$fU = -\lambda V, fV = \lambda U, \tag{26}$$

$$uof = \lambda v, vof = -\lambda U, \tag{27}$$

$$u\,(U) = -1 - \lambda^2, u\,(V) = v\,(U) = 0, v\,(V) = -1 - \lambda^2, \tag{28}$$

$$g\,(fX,\ fY) = -g\,(X,\ Y) - u\,(X)\,u\,(Y) - v\,(X)\,v(Y), \tag{29}$$

$$g\,(X,\ fY) = -g\,(fX,\ Y)\,,\ g\,(X,\ U) = u(X), g\,(X,\ V) = v(X) \tag{30}$$

for all for $X, Y \in TM$, where

$$\lambda = \eta(N). \tag{31}$$

Thus, we see that every transversal hypersurface of an almost hyperbolic contact metric manifold endowed with a semi symmetric non metric connection also admits a hyperbolic (f, g, u, v, λ)-structure. Next we find relation between the induced almost product structure (F, G) and the induced hyperbolic (f, g, u, v, λ)-structure on the transversal hypersurface of an almost hyperbolic contact metric manifold endowed with a semi symmetric non metric connection. In fact, we have the following.

Theorem 2.3. *Let M be a transversal hypersurface of an almost hyperbolic contact metric manifold $\overline{M}$ endowed with a semi symmetric non metric connection equipped with almost hyperbolic contact metric structure (ϕ, ξ, η, g) and induced almost product structure (F, G). Then we have*

$$\lambda\omega = u, \tag{32}$$

$$F = f - \frac{1}{\lambda} u \otimes V, \tag{33}$$

$$FU = \frac{1}{\lambda} V, \tag{34}$$

$$uoF = uof = \lambda v, \tag{35}$$

$$FV = fV = \lambda U, \tag{36}$$

$$uoF = \frac{1}{\lambda} u. \tag{37}$$

Proof.

$$\phi X = FX + \omega(X)\xi,$$
$$\xi = V + \lambda\ N,$$
$$\phi X = FX + \omega\left(X\right)V + \lambda\omega\left(X\right)N, \tag{38}$$
$$\phi X = fX + u(X)N. \tag{39}$$

From equation (38) and (39) we have

$$\lambda\omega X = u(X),\ \omega\left(X\right) = \frac{1}{\lambda}u(X),$$
$$FX = fX - \omega\left(X\right)V,$$
$$FX = fX - \frac{1}{\lambda}u(X)V,$$
$$F = f - \frac{1}{\lambda}u \otimes V,$$

which is equation (33).

$$\left(uoF\right)\left(X\right) = \left(uof\right) - \frac{1}{\lambda}u\left(X\right)u(V),\ u\left(V\right) = 0,$$
$$uoF = uof = \lambda v,$$

which is equation (35).

$$FU = fV - \frac{1}{\lambda}u(v)V,$$
$$FU = -\lambda V - \frac{1}{\lambda}\left(-1-\lambda^2\right)V = \frac{1}{\lambda}V,$$
$$FU = \frac{1}{\lambda}V,$$

which is equation (34).

$$\left(uoF\right)\left(X\right) = \left(uof\right)\left(X\right) - \frac{1}{\lambda}u\left(X\right)u(V) =$$
$$= \left(uof\right)\left(X\right) - \frac{1}{\lambda}u\left(X\right)\left(-1-\lambda^2\right) =$$
$$= -\lambda u\left(X\right) + \frac{1}{\lambda}u\left(X\right) + \lambda u(X) =$$
$$= \frac{1}{\lambda}u\left(X\right),$$
$$uoF = \frac{1}{\lambda}u,$$
$$FV = fV - \frac{1}{\lambda}u\left(V\right)V = fV = \lambda U,$$

which is equation (36) here equations (26), (27), (28), (29), (30), (31) are used. □

Lemma 1. *Let M be a transversal hypersurface with hyperbolic $(f, g,\ u, v, \lambda)$-structure of an almost hyperbolic contact metric manifold $\overline{M}$ endowed with a semi symmetric non metric connection. Then*

$$\left(\overline{\nabla}_X\phi\right)Y = \left(\nabla_X f\right)Y - u\left(Y\right)HX + \lambda X u\left(Y\right) + h\left(X,Y\right)N + h\left(X, fY\right)N + \left(\nabla_X u\right)(Y)N, \tag{40}$$
$$\overline{\nabla}_X\xi = \nabla_X V - \lambda HX + \lambda^2 X + \left(h\left(X,V\right) + X(\lambda)\right)N, \tag{41}$$
$$\left(\overline{\nabla}_X\phi\right)N = -\nabla_X U + fHX - f\lambda X - \left[h\left(X,U\right) + \lambda u(X) - \mu(HX)\right]N, \tag{42}$$
$$\left(\overline{\nabla}_X\eta\right)Y = \left(\nabla_X v\right)Y + h\left(X,Y\right)\lambda \tag{43}$$

for all $X, Y \in TM$.

The proof is straight forward and hence omitted.

3. Transversal hypersurfaces of cosymplectic hyperbolic manifold endowed with semi symmetric non metric connection

Trans-Sasakian structures of type $(\alpha, 0)$ are called α-Sasakian and trans-Sasakian structures of type $(0, \beta)$ are called β-Kenmotsu structures. Trans-Sasakian structures of type $(0, 0)$ are called cosymplectic structures.

Theorem 3.1. *Let M be a transversal hypersurfaces with hyperbolic (f, g, u, v, λ)-structure of a hyperbolic cosymplectic manifold $\overline{M}$ endowed with a semi symmetric non metric connection. Then*

$$(\nabla_X f)\,Y = u\,(Y)\,HX - \lambda X u(Y), \tag{44}$$

$$(\nabla_X u)\,Y = -h\,(X, Y) - h\,(X, fY)\,, \tag{45}$$

$$\nabla_X V = \lambda HX - \lambda^2 X, \tag{46}$$

$$h\,(X, V) = -X\lambda, \tag{47}$$

$$\nabla_X U = fHX - f\lambda X \quad \text{and} \quad h\,(X, U) = u\,(HX) - \lambda u(X), \tag{48}$$

$$(\nabla_X v)\,Y = -h(X, Y)\lambda, \tag{49}$$

for all $X, Y \in TM$.

Proof. Using (7), (20), (23) in (40), we obtain

$$(\nabla_X f)\,Y - u\,(Y)\,HX + \lambda X u\,(Y) + h\,(X, Y)\,N + h\,(X, fY)\,N + (\nabla_X u)\,(Y)\,N = 0.$$

Equating tangential and normal parts in the above equation, we get (44) and (45) respectively. Using (8) and (23) in (41), we have

$$\nabla_X V - \lambda HX + \lambda^2 X + (h\,(X, V) + X(\lambda))\,N = 0.$$

Equating tangential and normal parts we get (46) and (47) respectively. Using (7), (22) and (23) in (42).
Using (7), (22) and (23) in (42) and equating tangential, we get (48). In the last (49) follows from (43). □

Theorem 3.2. *If M be a transversal hypersurface with hyperbolic $(f, g,\ u, v, \lambda)$-structure of the hyperbolic cosymplectic manifold endowed with a semi symmetric non metric connection, then the 2-form Φ on M is given by*

$$\Phi\,(\mathrm{X},\ \mathrm{Y}) = \mathrm{g}(\mathrm{X},\ \mathrm{fY})$$

is closed.

Proof. From (44) we get

$$(\nabla_X \Phi)\,(Y,\ Z) = h\,(X,\ Y)\,u\,(Z) - h\,(X,\ Z)\,u(Y),$$

$$(\nabla_X \Phi)\,(Y,\ Z) + (\nabla_Y \Phi)\,(Z,\ X) + (\nabla_Z \Phi)\,(X,\ Y) = 0.$$

Hence the theorem is proved. □

Theorem 3.3. *If M is a transversal hypersurface with almost product semi Riemannian structure (F,G) of a hyperbolic cosymplectic manifold endowed with a semi symmetric non metric connection, then the 2-form Ω on M is given by*

$$\Omega(X,Y) = G(X, fY)$$

is closed.
Using (44), we calculate the Nijenhuis tensor

$$[F,F] = (\nabla_{FX}\mathrm{F})\,Y - (\nabla_{FY}\mathrm{F})\,X - F(\nabla_X\mathrm{F})\,Y + F(\nabla_Y\mathrm{F})\,X$$

and find that $[F,\ F] = 0$.

Therefore, in view of theorem (45), we have

Theorem 3.4. *Every transversal hypersurface of a trans hyperbolic cosymplectic manifold endowed with a semi symmetric non metric connection, admits product structure.*

4. Transversal hypersurfaces of trans hyperbolic Sasakian manifold endowed with semi symmetric non metric connection

Theorem 4.1. *Let M be a transversal hypersurface with hyperbolic (f,g,u,v,λ)-structure of a trans hyperbolic Sasakian manifold $\overline{M}$ endowed with a semi symmetric non metric connection. Then*

$$(\nabla_X f)\,Y = \alpha\{g(X,Y)\,V - \eta(Y)X\} + \tag{50}$$
$$+\beta\{g(fX,Y)\,V - \eta(Y)fX\} - \eta(Y)\,fX + u(Y)\,HX - \lambda X u(Y),$$

$$(\nabla_X u)\,Y = \alpha\lambda g(X,Y) + \beta\{\lambda g(fX,Y) - u(X)\,\eta(Y)\} - u(X)\,\eta(Y) - h(X,Y) - h(X,fY), \tag{51}$$

$$\nabla_X V = \alpha fX + \beta(X + \eta(X)V) - X + \lambda HX - \lambda^2 X, \tag{52}$$

$$h(X,V) = \alpha u(X) + \beta\lambda\eta(X) - X\lambda, \tag{53}$$

$$\nabla_X U = \alpha\lambda X + \beta(\lambda fX - u(X)\,V) + fHX, \tag{54}$$

$$h(X,U) = u(HX) \tag{55}$$

for all $X, Y \in TM$.

Proof. Using (7), (21), (23) in (40), we obtain

$$(\nabla_X f)\,Y - u(Y)\,HX + \lambda X u(Y) + h(X,Y)\,N + h(X,fY)\,N + (\nabla_X u)(Y) =$$
$$= \alpha\{g(X,Y)\,V + \lambda g(X,Y)\,N - \eta(Y)\,X\} +$$
$$+\beta\{g(fX,Y)\,V + \lambda g(fX,Y)\,N - \eta(Y)\,fX - u(X)\,\eta(Y)\,N\} - \eta(Y)\,fX - u(X)\,\eta(Y)\,N.$$

Equating tangential and normal parts in the above equation, we get (50) and (51) respectively. Using (8) and (23) in (41), we have

$$\nabla_X V - \lambda HX + \lambda^2 X + (h(X,V) + X(\lambda))\,N = \alpha fX + \alpha u(X)\,N + \beta(X + \eta(X)\,V + \lambda\eta(X)N) - X.$$

Equating tangential and normal parts we get (52) and (53) respectively. Using (7), (22) and (23) in (42) and equating tangential parts, we get (54) in the last (55) follows from (43). □

Theorem 4.2. *If M be a transversal hypersurface with hyperbolic $(f,g,\ u,v,\lambda)$ structure of $a(\alpha,\ o)$ trans hyperbolic Sasakian manifold endowed with a semi symmetric non metric connection, then the 2-form Φ on M is given by*

$$\Phi(X,Y) = g(X, fY)$$

is closed.

Proof. From (50) we get

$$(\nabla_X \Phi)(Y,\ Z) = -\alpha\{g(X,Y)v(Z) - g(X,Z)v(Y)\} - \beta\{g(fX,Y)v(Z) - g(fX,\ Z)v(Y)\} +$$
$$+h(X,Y)u(Z) - h(X,Z)u(Y),$$

which gives

$$(\nabla_X \Phi)(Y,\ Z) + (\nabla_Y \Phi)(Z,\ X) + (\nabla_Z \Phi)(X,\ Y) =$$
$$= 2\beta(\Phi(X,\ Y)\eta(Z) + \Phi(Y,\ Z)\eta(X) + \Phi(Z,X)\eta(Y)).$$

If $\beta = 0$, then

$$(\nabla_X \Phi)(Y,\ Z) + (\nabla_Y \Phi)(Z,\ X) + (\nabla_Z \Phi)(X,\ Y) = 0$$

that is

$$d\Phi = 0.$$

Hence the theorem is proved. □

Theorem 4.3. *If M is a transversal hypersurface with almost product semi Riemannian structure (F,G) of a $(\alpha,0)$ trans hyperbolic Sasakian manifold endowed with a semi symmetric non metric connection. Then 2-form Ω on M is given by*

$$\Omega\ (X,\ Y) = G(X,\ FY)$$

is closed.
Using (50), we calculate the Nijenhuis tensor

$$[F,F] = (\nabla_{FX}\mathrm{F})Y - (\nabla_{FY}\mathrm{F})X - F(\nabla_X\mathrm{F})Y + F(\nabla_Y\mathrm{F})X$$

and find that $[F,F] = 0$.

Therefore, in view of theorem 4.2, we have

Theorem 4.4. *Every transversal hypersurface of a trans hyperbolic Sasakian manifold endowed with a semi symmetric non metric connection admits a product structure.*

Example: Let us consider a 3-dimensional manifold $M = \{(x,y,z) \in R^3 : z \neq 0\}$, where $(x,\ y,z)$ are the standard coordinates in R^3.

Let $e_1 = e^z\left(\frac{\partial}{\partial x} + y\frac{\partial}{\partial z}\right)$, $e_2 = e^z\frac{\partial}{\partial y}$ and $e_3 = \frac{\partial}{\partial z}$, which are linearly independent vector fields at each point of M. Define a semi-Reimannian metric g on M

$$g(e_1,\ e_3) = g(e_2,\ e_3) = g(e_1,\ e_2) = 0,$$

$$g(e_1,\ e_1) = g(e_2,\ e_2) = g(e_3,\ e_3) = \pm 1.$$

Let η be the 1-form defined by $\eta(Z) = g(Z,\ e_3)$, for any $Z \in \Gamma(TM)$ and ϕ be the tensor field of type $(1,1)$ defined by $\phi e_1 = e_2$, $\phi e_2 = -e_1$, $\phi e_3 = 0$. Then by applying linearity of ϕ and g, we have

$$\eta(e_3) = -1,\ \phi^2 Z = Z + \eta(Z)e_3,\ g(\phi Z, \phi U) = -g(Z,U) - \eta(Z)\eta(U)$$

for any $Z, U \in \Gamma(TM)$. Hence for $e_3 = \xi$ defines an-almost hyperbolic contact metric structure on $\overline{M}$.

Let $\overline{\nabla}$ be the Levi-Civita connection with respect to g and R be the curvature tensor of type $(1,\ 3)$. Then we have

$$[e_1,\ e_2] = \left[ye^z e_2 - e^{2z}e_3\right], [e_1,\ e_3] = -e_1, [e_2,\ e_3] = -e_2.$$

By using Koszul's formula for the Levi-Civita connection with respect to g, we obtain

$$\overline{\nabla}_{e_1}e_3 = -e_1 + \frac{1}{2}e^{2z}e_2, \overline{\nabla}_{e_2}e_3 = -e_2 - \frac{1}{2}e^{2z}e_1, \overline{\nabla}_{e_3}e_3 = 0,$$

$$\overline{\nabla}_{e_1}e_2 = -\frac{1}{2}e^{2z}e_3, \overline{\nabla}_{e_2}e_2 = e_3 + ye^{z}e_1, \overline{\nabla}_{e_3}e_2 = -\frac{1}{2}e^{2z}e_1,$$

$$\overline{\nabla}_{e_1}e_1 = e_3, \overline{\nabla}_{e_2}e_1 = -ye^{z}e_2 + \frac{1}{2}e^{2z}e_3, \overline{\nabla}_{e_3}e_1 = \frac{1}{2}e^{2z}e_2.$$

Now, for $\xi = e_3$, above results satisfy

$$\overline{\nabla}_X\xi = \alpha\phi X + \beta\left\{X + \eta(X)\xi\right\} - X$$

with $\alpha = \frac{1}{2}e^{2z}$ and $\beta = 0$. Consequently $M\left(\phi, \xi, \eta, g\right)$ is a 3-dimensional transversal hypersurfaces of almost hyperbolic contact manifolds endowed with a semi symmetric non metric connection.

5. Conclusion

Here our ambition is to introduce transversal hypersurfaces of almost hyperbolic contact manifolds endowed with a semi symmetric non metric connection and find out some new results. It is proved that transversal hypersurfaces of almost hyperbolic contact manifold endowed with a semi symmetric non metric connection admits an almost product structure and each transversal hypersurfaces of almost hyperbolic contact metric manifold endowed with a semi symmetric non metric connection admits an almost product semi-Riemannian structure. Further I have tried to show that the fundamental 2-form on the transversal hypersurfaces of cosymplectic hyperbolic manifold and $(\alpha,\ 0)$ trans hyberbolic Sasakian manifold endowed with a semi symmetric non metric connection with hyperbolic $(f, g,\ u, v, \lambda)$-structure are closed. It is also proved that transversal hypersurfaces of trans hyperbolic contact manifold endowed with a semi symmetric non metric connection admits a product structure. It is applicable for different connections.

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