TWMS J. Pure Appl. Math., V.5, N.1, 2014, pp.36-42

EXISTENCE AND UNIQUENESS OF SOLUTION OF NONSTATIONARY BOLTZMANN'S MOMENT SYSTEM EQUATIONS IN THIRD APPROXIMATION

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Abstract. In the article it is proved the existence and uniqueness of the solution of the initial and boundary value problem for the nonstationary nonlinear one-dimensional Boltzmann's moment system equations in third approximation in space of functions, continuous in time and square summable by spatial variable.

Keywords: Boltzmann's moment system equations, nonlinear, kinetic theory.

AMS Subject Classification: 35Q20.

1. Introduction

In case of one-atom gas any macroscopical system in the process of its evolution to an equilibrium state passes 3 stages: initial transition period – described in terms of full function distribution of system, the kinetic period – by means of one-partial distribution function, the hydrodynamic period – by means of the five first moments of distribution function. Boltzmann's moment system equations are intermediate between kinetic and hydrodynamic levels of the description of the state of the rarefied gas. These five equations form non closed system, as contain 13 unknowns. To close this system of equations we may express stress tenzor and heat flux through pressure, temperature and so on. Thus, from the equations corresponding to above mentioned laws it is possible to receive Euler's equations, Navier-Stokes equations, Burnett equations and others. In general, the solution of any problem for nonlinear Boltzmann's moment system equations presents more complexity than the solution of Navier-Stokes equations.

An approximation strategy in kinetic theory is given by Grad's [3] moment method based on Hilbert expansion of the distribution function in Hermite polynomials. The method is described in Grad (1949) [3]. Differential part of Grad's system contains as coefficients such unknown hydro-dynamical parameters like density, temperature, average speed. The statement of boundary value problems for Grad's system became difficult.

In works [1], [12] there had been received the moment systems for spatially-homogeneous Boltzmann equation, and the conditions of representability of solution of the spatially-homogeneous Boltzmann equation in form of Henri Poincare series. Let us notice that the method offered in work [1] (application of the Fourier transform on a velocity variable in an isotropic case) had strongly simplified the integral of collisions and, hence, calculation of the moments from integral of collisions. In work [12] had been generalized the results of work [1] for a case of anisotropic dispersion.

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Manuscript received October 2012.

In work [9] had been received the system of moment equations for the spatially-non- homogeneous Boltzmann equation, which is distinct from Grad's system, by expansion of distribution function of particles by eigenfunctions of the linearized collision operator. Thus the differential part of the moment system has appeared to be linear, and the moments of the nonlinear collision operator are sign-non-defined square forms.

In work [10] it is proved the existence of the global solution in time of the initial and boundary value problem for one-dimensional nonlinear Boltzmann's moment system equations in second approximation in space of functions, continuous in time and summable by spatial variable, more exactly, in space of functions $C\left([0,T]\,;L^1 lnL^1\right)$. Furthermore, essentially had been used some inner properties, inherent to the system of moment equations in second approximation, namely analogues of mass conservation law and Boltzmann's H-theorem. Therefore studying of an initial or initial-boundary value problem for each approximation of Boltzmann's moment system equations represents the big interest.

In this article we consider the initial and boundary value problem for one-dimensional non-stationary nonlinear Boltzmann's moment system equations in third approximation and prove the existence of the solution in space of functions, continuous in time and square summable by spatial variable.

2. Initial and boundary value problem for the nonstationary nonlinear onedimensional Boltzmann's moment system equations in third approximation

We consider one-dimensional nonstationary nonlinear Boltzmann's moment system equations in k-th approximation [8]

$$\frac{\partial\varphi_{nl}}{\partial t}+\frac{1}{\alpha}\frac{\partial}{\partial x}\left\{l\left[\sqrt{\frac{2(n+l+\frac{1}{2})}{(2l-1)\,(2l+1)}}\varphi_{n,l-1}-\sqrt{\frac{2(n+l)}{(2l-1)\,(2l+1)}}\varphi_{n+1,l-1}\right]+\right.$$
$$\left.+(l+1)\left[\sqrt{\frac{2(n+l+\frac{3}{2})}{(2l+1)\,(2l+3)}}\varphi_{n,l+1}-\sqrt{\frac{2n}{(2l+1)\,(2l+3)}}\varphi_{n-1,l+1}\right]\right\}=I_{nl}, \qquad (1)$$
$$2n+l=0,1,\ldots,k,$$

where I_{nl} are the moments of nonlinear collision operator which expressed in terms of coefficients of Talmi and Klebsh-Gordon [5], [6], and have the form

$$I_{nl}=\sum\langle N_3L_3n_3l_3:l/nloo:l\rangle\,\langle N_3L_3n_3l_3:l/n_1l_1n_2l_2:l\rangle\,(l_1ol_2o/lo)\,V^{(l_3)}\varphi_{n_1,l_1}\varphi_{n_2,l_2}.$$

For generalized coefficients of Talmi exists a table [5] for each value of quantum number $\xi=2n+l$ from 0 to 6. Moreover, it is been created the program on IBM for calculation of generalized coefficients of Talmi.

If in (1) 2n+l takes values from 0 to 3, then we get the Boltzmann's moment system equations in third approximation. We write Boltzmann's moment system equations in third approximation in an expanded form

$$\frac{\partial\varphi_{00}}{\partial t}+\frac{1}{\alpha}\frac{\partial\varphi_{01}}{\partial x}=0,$$
$$\frac{\partial\varphi_{02}}{\partial t}+\frac{1}{\alpha}\frac{\partial}{\partial x}\left(\frac{2}{\sqrt{3}}\varphi_{01}+\frac{3}{\sqrt{5}}\varphi_{03}-\frac{2\sqrt{2}}{\sqrt{15}}\varphi_{11}\right)=I_{02},$$
$$\frac{\partial\varphi_{10}}{\partial t}+\frac{1}{\alpha}\frac{\partial}{\partial x}\left(-\sqrt{2/3}\varphi_{01}+\sqrt{5/3}\varphi_{11}\right)=0,$$

$$\frac{\partial\varphi_{01}}{\partial t}+\frac{1}{\alpha}\frac{\partial}{\partial x}\left(\varphi_{00}+\frac{2}{\sqrt{3}}\varphi_{02}-\sqrt{2/3}\varphi_{10}\right)=0,$$
$$\frac{\partial\varphi_{03}}{\partial t}+\frac{1}{\alpha}\frac{\partial}{\partial x}\frac{3}{\sqrt{5}}\varphi_{02}=I_{03},$$
$$\frac{\partial\varphi_{11}}{\partial t}+\frac{1}{\alpha}\frac{\partial}{\partial x}\left(-\frac{2\sqrt{2}}{\sqrt{15}}\varphi_{02}+\sqrt{5/3}\varphi_{10}\right)=I_{11}, \qquad (2)$$

where $I_{02}=(\sigma_2-\sigma_0)\left(\varphi_{00}\varphi_{02}-\varphi_{01}{}^2/\sqrt{3}\right)/2$,

$$I_{03}=\frac{1}{4}\left(\sigma_3-3\sigma_1-4\sigma_0\right)\varphi_{00}\varphi_{03}+\frac{1}{4\sqrt{5}}(2\sigma_1+\sigma_0-3\sigma_3)\varphi_{01}\varphi_{02},$$
$$I_{11}=(\sigma_1-\sigma_0)\left(\varphi_{00}\varphi_{11}+\frac{1}{2}\sqrt{5/3}\varphi_{10}\varphi_{01}-\frac{2}{\sqrt{15}}\varphi_{01}\varphi_{02}\right),$$

σ_0,σ_1, σ_2,σ_3 are constants.

First, third and fourth equations of the system (2) correspond to the mass conservation law, momentum conservation law and energy conservation law correspondingly.

We introduce following vectors and matrices

$$U=(\varphi_{00},\varphi_{02},\varphi_{10},\varphi_{01},\varphi_{03},\varphi_{11})',$$
$$A=\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 2/\sqrt{3} & 3/\sqrt{5} & -2\sqrt{2/15}\\ 0 & 0 & 0 & -\sqrt{2/3} & 0 & \sqrt{5/3}\\ 1 & 2/\sqrt{3} & -\sqrt{2/3} & 0 & 0 & 0\\ 0 & 3/\sqrt{5} & 0 & 0 & 0 & 0\\ 0 & -2\sqrt{2/15} & \sqrt{5/3} & 0 & 0 & 0 \end{pmatrix},$$
$$\Lambda=diag\left(0,\ -\lambda_{02},0,0,-\lambda_{03},-\lambda_{11}\right),$$
$$I(U,U)=\left(0,\ I_{02},0,0,I_{03},I_{11}\right)'.$$

We write the system of equations (2) in vector-matrix form

$$\frac{\partial U}{\partial t}+\frac{1}{\alpha}A\frac{\partial U}{\partial x}+\Lambda U=I\left(U,U\right). \qquad (3)$$

Eigenvalues and orthonormalized eigenvectors of matrix A:

$$-\sqrt{3+\sqrt{6}},\ -1,\ -\sqrt{3-\sqrt{6}},\ \sqrt{3-\sqrt{6}},\ 1,\ \sqrt{3+\sqrt{6}};$$
$$\left(\frac{1}{2}\sqrt{\frac{3}{2}}\frac{\sqrt{6}+2}{\left(\sqrt{6}+3\right)^{3/2}},\ \frac{\sqrt{3+\sqrt{6}}}{3\sqrt{2}},\ -\frac{1}{2}\frac{5+2\sqrt{6}}{\left(\sqrt{6}+3\right)^{3/2}},\ -\frac{1}{2}\sqrt{\frac{3}{2}}\frac{\sqrt{6}+2}{\sqrt{6}+3},\ -\frac{1}{\sqrt{10}},\ \sqrt{\frac{3}{20}}\right)';$$
$$\left(0,\ -\frac{1}{\sqrt{6}},\ -\frac{1}{\sqrt{3}},\ 0,\ \sqrt{\frac{3}{10}},\ \frac{1}{\sqrt{5}}\right)';$$
$$\left(-\frac{1}{2}\sqrt{\frac{3}{2}}\frac{\sqrt{6}-2}{\left(3-\sqrt{6}\right)^{3/2}},\ \frac{\sqrt{3-\sqrt{6}}}{3\sqrt{2}},\ -\frac{1}{2}\frac{5-2\sqrt{6}}{\left(3-\sqrt{6}\right)^{3/2}},\ -\frac{1}{2}\sqrt{\frac{3}{2}}\frac{\sqrt{6}-2}{\sqrt{6}-3},\ -\frac{1}{\sqrt{10}},\ \sqrt{\frac{3}{20}}\right)';$$
$$\left(\frac{1}{2}\sqrt{\frac{3}{2}}\frac{\sqrt{6}-2}{\left(3-\sqrt{6}\right)^{3/2}},\ -\frac{\sqrt{3-\sqrt{6}}}{3\sqrt{2}},\ \frac{1}{2}\frac{5-2\sqrt{6}}{\left(3-\sqrt{6}\right)^{3/2}},\ -\frac{1}{2}\sqrt{\frac{3}{2}}\frac{\sqrt{6}-2}{\sqrt{6}-3},\ -\frac{1}{\sqrt{10}},\ \sqrt{\frac{3}{20}}\right)';$$
$$\left(0,\ \frac{1}{\sqrt{6}},\ \frac{1}{\sqrt{3}},\ 0,\ \sqrt{\frac{3}{10}},\ \frac{1}{\sqrt{5}}\right)';$$

$$\left(-\frac{1}{2}\sqrt{\frac{3}{2}}\frac{\sqrt{6}+2}{\left(\sqrt{6}+3\right)^{3/2}},\ -\frac{\sqrt{3+\sqrt{6}}}{3\sqrt{2}},\ \frac{1}{2}\frac{5+2\sqrt{6}}{\left(\sqrt{6}+3\right)^{3/2}},\ -\frac{1}{2}\sqrt{\frac{3}{2}}\frac{\sqrt{6}+2}{\sqrt{6}+3},\ -\frac{1}{\sqrt{10}},\ \sqrt{\frac{3}{20}}\right)'.$$

Let's consider the vector

$$\psi=(\psi_1,\psi_2,\psi_3,\psi_4,\psi_5,\psi_6)'.$$

We denote as B the matrix, columns of which form the eigenvectors of matrix A. System of equations (3) in canonical view we write as

$$\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\\ \psi_5\\ \psi_6\end{pmatrix}+\frac{1}{\alpha}\frac{\partial}{\partial x}\begin{pmatrix}-\sqrt{3+\sqrt{6}}\psi_1\\ -\psi_2\\ -\sqrt{3-\sqrt{6}}\psi_3\\ \sqrt{3-\sqrt{6}}\psi_4\\ \psi_5\\ \sqrt{3+\sqrt{6}}\psi_6\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{3+\sqrt{6}}}{3\sqrt{2}}J_{02}(\psi,\psi)-\frac{1}{\sqrt{10}}J_{03}(\psi,\psi)+\sqrt{\frac{3}{20}}J_{11}(\psi,\psi)\\ -\frac{1}{\sqrt{6}}J_{02}(\psi,\psi)+\sqrt{\frac{3}{10}}J_{03}(\psi,\psi)+\frac{1}{\sqrt{5}}J_{11}(\psi,\psi)\\ \frac{\sqrt{3-\sqrt{6}}}{3\sqrt{2}}J_{02}(\psi,\psi)-\frac{1}{\sqrt{10}}J_{03}(\psi,\psi)+\sqrt{\frac{3}{20}}J_{11}(\psi,\psi)\\ -\frac{\sqrt{3-\sqrt{6}}}{3\sqrt{2}}J_{02}(\psi,\psi)-\frac{1}{\sqrt{10}}J_{03}(\psi,\psi)+\sqrt{\frac{3}{20}}J_{11}(\psi,\psi)\\ \frac{1}{\sqrt{6}}J_{02}(\psi,\psi)+\sqrt{\frac{3}{10}}J_{03}(\psi,\psi)+\frac{1}{\sqrt{5}}J_{11}(\psi,\psi)\\ -\frac{\sqrt{3+\sqrt{6}}}{3\sqrt{2}}J_{02}(\psi,\psi)-\frac{1}{\sqrt{10}}J_{03}(\psi,\psi)+\sqrt{\frac{3}{20}}J_{11}(\psi,\psi)\end{pmatrix},$$

$$x\in[-a;a],\ t>0, \tag{4}$$

where $U=B\psi$, and $J_{02}(\psi,\psi)$, $J_{03}(\psi,\psi)$, $J_{11}(\psi,\psi)$ are square forms, in which instead of U we set ψ.

System of equations (4) is the nonlinear hyperbolic system of equations.

Let's set the following initial and boundary conditions for the system of equations (4):

$$\psi_i(0,x)=\psi_i^0(x),\ x\in[-a;a],\ i=\overline{1,6} \tag{5}$$

$$\psi_i(t,-a)=\psi_i(t,a),\quad i=\overline{1,6},\quad t>0. \tag{6}$$

We denote by $D=diag\frac{1}{\propto}\left(-\sqrt{3+\sqrt{6}},\ -1,\ -\sqrt{3-\sqrt{6}},\ \sqrt{3-\sqrt{6}},\ 1,\ \sqrt{3+\sqrt{6}}\right)$ a diagonal matrix and

$$J(\psi,\psi)=\left(\frac{\sqrt{3+\sqrt{6}}}{3\sqrt{2}}J_{02}(\psi,\psi)-\frac{1}{\sqrt{10}}J_{03}(\psi,\psi)+\sqrt{\frac{3}{20}}J_{11}(\psi,\psi),\dots,\right.$$

$$\left.-\frac{\sqrt{3+\sqrt{6}}}{3\sqrt{2}}J_{02}(\psi,\psi)-\frac{1}{\sqrt{10}}J_{03}(\psi,\psi)++\sqrt{\frac{3}{20}}J_{11}(\psi,\psi)\right)'$$

the vector of square forms.

We rewrite initial and boundary value problem (4)-(6) in the form

$$\frac{\partial\psi}{\partial t}+D\frac{\partial\psi}{\partial x}=J(\psi,\psi),\ x\in[-a;a],\ t>0, \tag{7}$$

$$\psi|_{t=0}=\psi_0(x),\ x\in[-a;a], \tag{8}$$

$$\psi(t,-a)=\psi(t,a). \tag{9}$$

For the problem (7)-(9) we have following theorem.

Theorem 2.1. *If $\psi_0\in L^2[-a;a]$, then there exists such a T, that the problem (7)-(9) has in the domain [0,T]x$[-a;a]$ a unique solution belonging to $C\left([0,T];L^2[-a;a]\right)$, and*

$$\|\psi\|_{C([0,T];L^2[-a;a])}\le C_1\|\psi_0\|_{L^2[-a;a]}, \tag{10}$$

where C_1 is a constant independent of $\psi, T\sim \mathrm{O}\left(\|\psi_0\|^{-1}_{L^2[-a;a]}\right)$.

Proof. Let $\psi_0 \in L^2[-a;a]$. We prove (10). We take the inner product of both sides of (7) with ψ and integrate over $[-a;a]$:

$$\int_{-a}^{a} \left(\left(\frac{\partial \psi}{\partial t} + D\frac{\partial \psi}{\partial x} \right), \psi \right) dx = \int_{-a}^{a} \left(J\left(\psi, \psi\right), \psi \right) dx.$$

Hence take into account the boundary condition (9) we have

$$\frac{1}{2}\frac{d}{dt}\int_{-a}^{a} \left(\psi, \psi\right) dx = \int_{-a}^{a} \left(J\left(\psi, \psi\right), \psi \right) dx. \tag{11}$$

We use a spherical representation [7] of the vector $\psi\left(t,x\right) = r\left(t\right)\omega(t,x)$, where $r\left(t\right) = \|\psi(t,.)\|_{L^2[-a;a]}$, $\omega(t,x)$ is a unit vector, i.e., $\|\omega\|_{L^2[-a;a]} = 1$. Substituting the value $\psi = r\omega$ into (11), we have that

$$\frac{1}{2}\frac{d}{dt}r^2(t) = r^3(t)\int_{-a}^{a} \left(J\left(\omega, \omega\right), \omega \right) dx.$$

Let us $Q\left(t\right) = \int_{-a}^{a} \left(J\left(\omega, \omega\right), \omega \right) dx$.

Let us notice, that $Q\left(t\right)$ is sign-non-defined function. Then we get the following problem

$$\frac{dr}{dt} = r^2 Q\left(t\right),\ t > 0, \tag{12}$$

$$r\left(0\right) = \|\psi_0\| \equiv \|\psi_0\|_{L^2[-a;a]}\ . \tag{13}$$

The solution of the problem (12)-(13) has the form

$$r\left(t\right) = \left\{ \left(\frac{1}{\|\psi_0\|} - \int_0^t Q(\tau)d\tau \right) \right\}^{-1}.$$

If $(t) \equiv \int_0^t Q(\tau)d\tau \leq 0\ \forall t$, then $r(t)$ is bounded $\forall t \in [0, +\infty)$. Let $R\left(t\right) > 0$. Denote by T_1 the moment of time at which

$$\frac{1}{\|\psi_0\|} - \int_0^{T_1} Q(\tau)d\tau = 0.$$

Then $r(t)$ is bounded $\forall t \in [0,T]$, where $T < T_1$, and $T_1 \sim \mathrm{O}\left(\|\psi_0\|^{-1}\right)$, as the integrand $Q(\tau)$ is bounded. Hence $\forall t \in [0,T]$ takes place a priori estimation (10).

Now we prove the existence of a solution of (7)-(9) with the help of Galerkin method. Let us $\{\omega_l(x)\}_{l=1}^{\infty}$ be a basis in the space $L^2[-a;a]$, where dimension of vector $\omega_l(x)$ is equal to dimension of vector ψ. For each m we define an approximate solution ψ_m of (7)-(9) as follows:

$$\psi_m = \sum_{l=1}^{m} c_{lm}(t)\omega_l(x), \tag{14}$$

$$\int_{-a}^{a} \left(\left(\frac{\partial \psi_m}{\partial t} + D\frac{\partial \psi_m}{\partial x}\right), \omega_i(x) \right) dx = \int_{-a}^{a} \left(\left(J\left(\psi_m, \psi_m\right)\right), \omega_i(x) \right) dx, i = \overline{1,m},\ t \in (0,T], \tag{15}$$

$$\psi_m|_{t=0} = \psi_{0m}\left(x\right),\ x \in [-a;a], \tag{16}$$

where ψ_{0m} is the orthogonal projection in L^2 of the function ψ_0 on the subspace, spanned by $\omega_1, \ldots \omega_m$. The coefficients $c_{lm}(t)$ are determined from the equations

$$\sum_{l=1}^{m} \left(\frac{dc_{lm}}{dt} \int_{-a}^{a} (\omega_l, \omega_i)\, dx - \int_{-a}^{a} \left(D\omega_l, \frac{d\omega_i}{dx} \right) dx \right) =$$

$$= \int_{-a}^{a} \left((J \left(\sum_{l=1}^{m} c_{lm}\omega_l, \sum_{l=1}^{m} c_{lm}\omega_l \right)), \omega_i(x) \right) dx, i = \overline{1,m},\ t \in (0,T], \tag{17}$$

$$c_{im}(0) = d_{im}, i = \overline{1,m}, \tag{18}$$

where d_{im} is the i-th component of ψ_{0m}.
We multiply (15) by $c_{im}(t)$ and sum over i from 1 to m:

$$\int_{-a}^{a} \left((\frac{\partial \psi_m}{\partial t} + D\frac{\partial \psi_m}{\partial x}), \psi_m \right) dx = \int_{-a}^{a} ((J(\psi_m, \psi_m)), \psi_m)\, dx.$$

With the help of the above arguments we now prove that $r_m(t)$, where $\psi_m(t,x) = r_m(t)\omega_m(t,x)$, is bounded in some time interval $[0, T_m]$, $T_m \approx \mathrm{O}\left(\| \psi_{0m} \|^{-1} \right)$, $T_m \geq T\ \forall m$, and

$$\| \psi_m \|_{C([0,T];L^2[-a;a])} \leq C_2 \| \psi_{0m} \|_{L^2[-a;a]} \leq C_2 \| \psi_0 \|_{L^2[-a;a]}, \tag{19}$$

where C_2 is constant and independent of m.

Then solvability of system equations (14)-(16) or (17)-(18) follows from estimation (19).

Thus, the sequence $\{\psi_m\}$ of approximate solutions of the problem (7)-(9) is uniformly bounded in $C\left([0,T]; L^2[-a;a]\right)$. Moreover, homogeneous system of equations $\tau E + D\xi$ with respect to τ, ξ has only trivial solution. Then it follows from results in [11], that $\psi_m \to \psi$ is weak in $C\left([0,T]; L^2[-a;a]\right)$ and $J(\psi_m,\psi_m) \to J(\psi,\psi)$ is weak in $C\left([0,T]; L^2[-a;a]\right)$ as $m \to \infty$. Further, it can be shown by the standard method that the limit element is a weak solution of the problem (7)-(9).

The theorem is proved. □

3. Conclusion

We proved the existence and uniqueness of the local solution of the initial and boundary value problem for the nonstationary nonlinear one-dimensional Boltzmann's moment system equations in third approximation. Interval of time, in which exists the solution of the problem depends on the norm of a vector of initial functions. If the norm of a vector of initial functions is small value then the interval of time in which exists the solution of the problem becomes large.

References

[1] Bobylev, A.V., (1975), The Fourier transform method in the theory of the Boltzmann equation for Maxwellian molecules, Dokl. Akad. Nauk SSSR, 225, pp.1041-1044; English transl. in Soviet Phys. Dokl., (1975), 20.

[2] Cercignani, C., (1975), Theory and Application of the Boltzmann Equation, Milano, Italy.

[3] Grad, H., (1949), Kinetic Theory of Rarefied Gases, Comm. Pure Appl. Math, 2, 331p.

[4] Kogan, M.N., (1967), Dynamic of Rarefied Gas, Moscow, Nauka, 440p.

[5] Moshinsky, M., (1960), The Harmonic Oscillator in Modern Physics: from Atoms to Quarks, New York – London - Paris, 152p.

[6] Neudachin, V.G., Smirnov, U.F., (1969), Nucleon Association of Easy Kernel, Moscow, Nauka.

[7] Pokhozhaev, S.I., (1979), On an approach to nonlinear equation, Dokl. Akad. Nauk SSSR, 247, pp.1327-1331; English transl. in Soviet Phys. Dokl., 20, (1979).

[8] Sakabekov, A., (1992), A mixed problem for onedimensional Boltzmann's moment system equations in odd approximation, Differential equations, 28(5), pp.892-990.

[9] Sakabekov, A., (1990), Deduction of the Boltzmann's Moment System Equations, Dep. in VINITI, N.1670-90, Alma-Ata, 56p.

[10] Sakabekov, A., Auzhani, D., (2011), About Solvability of the Initial and Boundary Value Problem for Onedimensional Nonlinear Boltzmann's Moment System Equations, Math. Journal, Almaty, N.3.

[11] Tartar, L., (1979), Compensated compactness and applications to partial differential equations, Heribi-Walt Symposium 2, Ed. R.J.Knops, Research Notes in Math., IT 59, pp.136-212.

[12] Vedenyapin, V.V., (1981), Anisotropic solution of the nonlinear Boltzmann equation for Maxwellian molecules, Dokl. Akad. Nauk SSSR, 256, pp.338-342; English transl. in Soviet Phys. Dokl., 26 (1981).

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