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SYNTHESIS OF THE QUAD-ROTOR CONTROL ALGORITHMS IN THE BASIC FLIGHT MODES

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Abstract. The synthesis of the quad-rotor control algorithms in the basic flight modes is considered. These include the basic modes of the horizontal quad-rotor flight: position control, cruise speed control and reference path tracking. The control algorithms are designed using traditional optimization procedures including also some specific optimization procedures likewise singular optimization problem solution for cruise speed control synthesis and introducing phase lead in the control algorithm in order to improve the path tracking performance. The efficiency of the proposed algorithms and the comparison of results obtained in this paper with results of other authors are illustrated by examples.

Keywords: quad-rotors, optimal control, linear control, path tracking, phase lag compensation.

AMS Subject Classification: 15A24, 93B35, 93C73.

1. Introduction

Traditional problems of stability, control and guidance continue to attract the attention of researches (see, for example, [2-12], where further references are given). Moreover the methods of solution of these problems are successfully applied for design of the practical systems especially the flight control systems [3- 8, 9-12]. Among various problems of the flight control, the problem of the quad-rotor motion control is very topical right now [2, 3, 12]. The most popular type of the quad-rotor control systems is very well known PID-control, which is successfully applied in many practical cases [11]. However the further enhancement of the quad-rotor control properties is possible via application of more sophisticated methods of control theory. Below we will show, that, using the dynamic model of quad-rotor [2, 3] and traditional optimization problem statement, it is possible to design linear controllers, which can compete effectively with nonlinear ones [9]. Application of these algorithms allows the purposeful changing of the performance functional parameters in order to obtain desirable dynamic behavior of the flight control system. The results of the computer simulation demonstrate significant improvement of the LQR-based quad-rotor flight control system performance in comparison with results obtained in [3]. On the basis of these results the problem of the reference path tracking by the quad-rotor is considered below [4, 8, 12]. Then the problem of the quad-rotor cruise speed control is considered (note, that such kind of problems preserve their actuality [16, 10]). The peculiarities of such problem are stated [1].

All theoretical results are illustrated by examples showing the efficiency of proposed algorithms.

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2. Dynamic model of the quad-rotor

The quad-rotor scheme is represented in Figure 1 (corresponds to Figure metric in [2]). Let $\xi = [x\ \ y\ \ z]^{'}$ is the radius-vector of the quad-rotor center of mass in some inertial frame $\{X, Y, Z\}$, ψ, θ, φ are the yaw, pitch and roll angles respectively, f_1 is the lifting force produced by the i-th motor M_i $(i = \overline{1, 4})$. Here and further prime denotes transposing. In accordance with [2, 3], the motion of this system is described by the following system of equations:

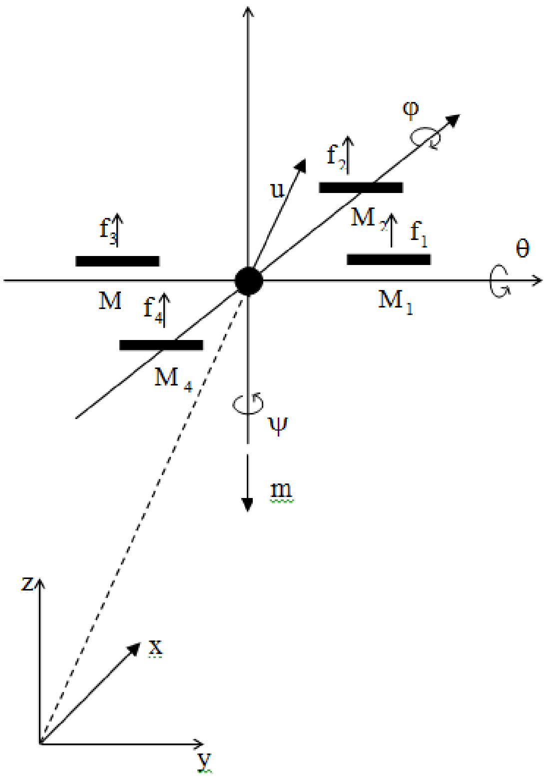

Figure 1.

$$m\frac{d^2x}{dt^2} = -u\sin\theta, \tag{1}$$

$$m\frac{d^2y}{dt^2} = u\cos\theta\sin\varphi, \tag{2}$$

$$m\frac{d^2z}{dt^2} = u\cos\theta\cos\varphi - mg, \tag{3}$$

$$\frac{d^2\psi}{dt^2} = \tilde{\tau}_\psi, \tag{4}$$

$$\frac{d^2\theta}{dt^2} = \tilde{\tau}_\theta, \tag{5}$$

$$\frac{d^2\varphi}{dt^2} = \tilde{\tau}_\varphi. \tag{6}$$

In equations (1) – (6) m stands for the mass of the quad-rotor, $g = 9,8/$ stands for the gravity acceleration, u, $\tilde{\tau}_\psi$, $\tilde{\tau}_\theta$, $\tilde{\tau}_\varphi$ are the control inputs, which are the functions of the lifting forces f_1. In [3, 4 2, 3] control input $u$is used for the quad-rotor altitude control and the control input $\tilde{\tau}_\psi$ allows the yaw angle stabilization. The control inputs $\tilde{\tau}_\theta$ and $\tilde{\tau}_\varphi$ are used for control of the pitch θ and roll φ angles as well as for control of the quad-rotor linear motion along the axes x and y respectively.

3. Altitude and yaw angle control algorithms

In accordance with [2, 3], the closed loop system of the quad-rotor altitude z control is defined by the following equation:

$$u = (r_1 + mg)\,\frac{1}{\cos\theta\cos\varphi}. \tag{7}$$

It is supposed in [2, 3] that $\cos\theta\cos\varphi \neq 0$. In equation (7) the $r_1(z, \dot{z})$ stands for the altitude PD-control law

$$r_1 = -a_{z_1}\dot{\dot{z}} - a_{z_2}(z - z_d) \tag{8}$$

where a_{z_1}, a_{z_2} are the differential and proportional gains (positive constants) and z_d is the reference (desired) altitude.
The yaw angle control law is the same:

$$\tilde{\tau}_\psi = -a_{\psi_1}\dot{\dot{\psi}} - a_{\psi_2}(\psi - \psi_d). \tag{9}$$

Supposing that$\cos\theta\cos\varphi \neq 0$, we have:

$$m\ddot{x} = -\,(r_1 + mg)\,\frac{\tan\theta}{\cos\varphi} \tag{10}$$

$$m\ddot{y} = (r_1 + mg)\tan\varphi \tag{11}$$

$$\ddot{\dot{z}} = \frac{1}{m}\left(a_{z_1}\dot{\dot{z}} - a_{z_2}(z - z_d)\right) \tag{12}$$

$$\ddot{\dot{\psi}} = -a_{\psi_1}\dot{\dot{\psi}} - a_{\psi_2}(\psi - \psi_d). \tag{13}$$

Coefficients a_{ψ_1}, a_{ψ_2},a_{z_1}, a_{z_2} in (12), (13) must be chosen from these systems asymptotic stability conditions. This circumstance in turn will provide the satisfaction of the conditions $\psi \to \psi_d$, $z \to z_d$.

4. Control of the coordinate pairs (φ, y) and (θ, x) [3, 4 2, 3]

It is noted in [2, 3] that the assumption about asymptotic stability permits to make the following conclusion: for sufficiently large value of the time period T it is possible to replace (10), (11) with following relations:

$$\frac{d^2x}{dt^2} = -g\frac{\tan\theta}{\cos\varphi} \tag{14}$$

$$.\frac{d^2y}{dt^2} = g\tan\varphi \tag{15}$$

Considering θ, φ as small angles and taking in account equations (6), (6), the following relations, which define these coordinates propagations, are given in [2, 3]:

$$\frac{d^2y}{dt^2} = g\varphi, \tag{16}$$

$$\frac{d^2\varphi}{dt^2} = \tilde{\tau}_\varphi, \tag{17}$$

$$\frac{d^2x}{dt^2} = -g\theta, \tag{18}$$

$$\frac{d^2\theta}{dt^2} = \tilde{\tau}_\theta. \tag{19}$$

It is recommended in [2, 3] to apply the following nonlinear algorithm [9] for stabilization of the coordinatesx, y:

$$\tilde{\tau}_\varphi = -\sigma_{\varphi 1}\left(\dot{\varphi} + \sigma_{\varphi 2}\left(\varphi + \dot{\varphi} + \sigma_{\varphi 3}\left(2\varphi + \dot{\varphi} + \frac{\dot{y}}{g} + \sigma_{\varphi 4}\left(\dot{\varphi} + 3\varphi + 3\frac{\dot{y}}{g} + \frac{y}{g}\right)\right)\right)\right), \tag{20}$$

$$\tilde{\tau}_\theta = -\sigma_{\theta 1}\left(\dot{\varphi} + \sigma_{\theta 2}\left(\theta + \dot{\theta} + \sigma_{\theta 3}\left(2\theta + \dot{\theta} + \frac{\dot{x}}{g} + \sigma_{\theta 4}\left(\dot{\theta} + 3\theta + 3\frac{\dot{x}}{g} + \frac{x}{g}\right)\right)\right)\right). \tag{21}$$

In these relations $\sigma_a(s)$ are nonlinear saturation functions [2, 3]:

$$\sigma_a(s) = \begin{cases} -a & s < -a; \\ s & -a \le s \le a; \\ a & s > a. \end{cases}$$

5. Comparison of linear and non-linear stabilization algorithms

This comparison was made in [3] by usage of examples. So, as an example, the system (16), (17) was considered. It may be represented in the following form:

$$\dot{p} = Ap + Bu, \tag{22}$$

$$p = \left[y,\ \dot{y},\ \varphi,\ \dot{\varphi}\right],$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}', u = \bar{\tilde{\tau}}\varphi.$$

For system (22) in [4 3] authors has synthesized optimal linear controller in accordance with the following performance index:

$$J = \int_0^\infty \left(p'Qp + u'Ru\right) dt. \tag{23}$$

In this performance index numerical values of the matrices Q and R were accepted in [4 3] as follows:

$$R = 1, Q = \begin{bmatrix} 1 & -2 & -4 & 6 \\ -2 & 4 & 8 & -12 \\ -4 & 8 & 16 & -24 \\ 6 & -12 & -24 & 36 \end{bmatrix}.$$

Applying standard procedure of the LQR-synthesis, the following numerical values of the controller gains (K) and the vector Z_{cl} of the eigenvalues of the closed loop system $(A - BK)$ were obtained:

$$K = \begin{bmatrix} 1 & 3.2848 & 29.3030 & 9.7266 \end{bmatrix},$$

$$Z_{cl} = [-5.2393, \quad -2.3946, \quad -1.6056, \quad -0.4870].$$

It is noted in [3] that open-loop linear system (16), (17) or (22) with this controller under following initial conditions $\dot{y}(0) = 0,\ y(0) = \pm 0,\ \dot{\varphi}(0) = 0,\ \varphi(0) = 0$ quickly returns to the equilibrium state (see Fig.2 in [3]). However in the case of the non-linear system (15), (17) this controller doesn't provide the stability of control system (see Fig.3 in [3]). Meanwhile the usage of the non-linear controller (20) allows stabilization of non-linear system under following initial conditions: $\dot{y}(0) = 0,\ y(0) = 200,\ \dot{\varphi}(0) = 0,\ \varphi(0) = 40$ (see Fig.5 in [3]).

Nevertheless comparing linear and non-linear controllers it is necessary to note, that in the linear case in accordance with the Fig.2 () [3] the $\varphi(t)$-plot two times during the transient process intersects the value$-\frac{\pi}{2}$. So the condition$\cos\varphi\cos\theta \neq 0$, imposed by authors earlier, is violated.

6. Control system optimization

It was mentioned above that the choice of the matrices R, Q for performance index (23) accepted in [3] is not optimal. In particular the linear controller obtained on the basis of such matrices R, Q leads to the violation of the condition $\cos\varphi\cos\theta \neq 0$ during the transient process. So it is expedient to modify weight matrices R and$Q$ in order to optimize the transient processes in the system.

Note that in the procedure of the LQR-synthesis in accordance with (22), (23) it is expedient to decrease the values of the state variables $\varphi, \dot{\varphi}$ and the control variable $\bar{\bar{\tau}}_\varphi$ during the transient process. Taking in account these considerations it is possible to choose the following values of the matrices R, Q entries:

$$R = 10^4, Q = \begin{bmatrix} 1 & -2 & -4 & 6 \\ -2 & 4 & 8 & -12 \\ -4 & 8 & 16 \cdot 10^4 & -24 \\ 6 & -12 & -24 & 36 \cdot 10^4 \end{bmatrix}. \qquad (24)$$

Optimizing (23) subject to (22) with new entries of matrices R, Q defined by (24), we obtain the following values of the controller gains (K) and the eigenvalues Z_{cl} of the closed-loop system$(A-BK)$:

$$K = \begin{bmatrix} 0.01 & 0.1083 & 5.5123 & 6.8574 \end{bmatrix}, \qquad (25)$$

$$Z_{cl} = [-5.9623, \quad -0.6702, \quad -0.1125 \pm 0.1090i].$$

The efficiency of this procedure we can estimate using example given in [4 3]. The following values of the initial conditions are accepted: $y(0) = 200 <$, $\dot{y}(0) = 0$, $\varphi(0) = 40^0$, $\dot{\varphi}(0) = 0$. The simulation results are represented in the Figures $2-5$.

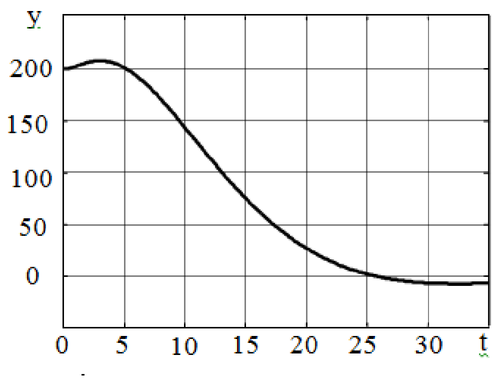

Figure 2.

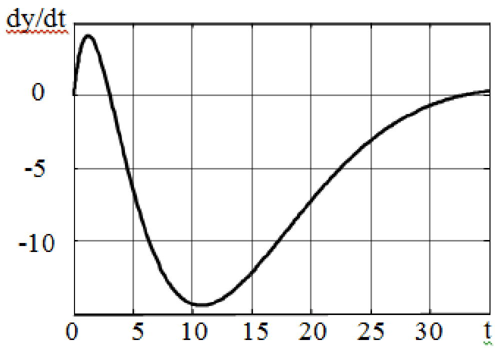

Figure 3.

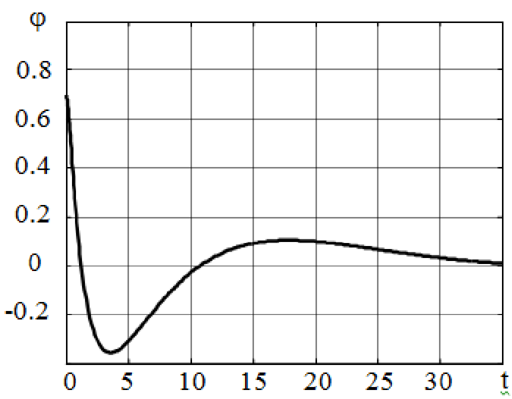

Figure 4.

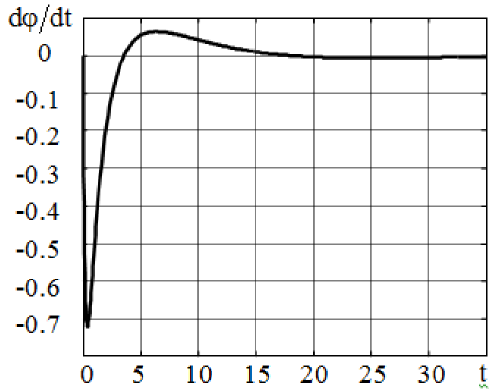

Figure 5.

Comparing these plots with the results represented in the Figure 5 of the reference [4 3], it is possible to conclude that controller considered in this item possesses certain advantages. Thus the $y(t)$- plot in the Figure 5a in [3] differs from zero essentially on the interval $0 - 125c$. Meanwhile in our case (see Figure 2) this interval equals $0 - 25c$.

Continuing further comparison of Figure 5b in [3] and Figure metricconverterProductID3 in3 in this paper it is possible to mark the following distinctions. If in Figure 5b [3] the $\dot{y}(t)$-plot abruptly changes practically from 0 to 200 m/sec during small period of time, then the corresponding plot in the Figure 3 of this paper demonstrates more smooth changing, when significant linear accelerations are absent and the maximal absolute value of the $\dot{y}(t)$-plot equals 15 m/sec. Note that if the controller (25) is used for control of the non-linear system (15), (17), then the simulation results, represented in the Figures 2 – 5, will change negligible.
This conclusion is derived from results shown in the Table 1.

Table 1.

	y(m)	dy/dt(m/sec)	φ(rad)	$d\varphi/dt$(rad/sec)
1	208,3	14,47	0,7	0,72
2	4,59	0,59	0,018	0,048

In the 1st raw of the table 1 the maximal absolute values of state space variables are given. Their time histories are shown in the Figures 2-5. In the 2nd raw the maximal values of the modules of differences between simulation results of system (16), (17), (25) and system (15), (17), (25) are given.
Therefore it could be stated that in the considered problems the efficiency of the linear controller with the proper choice of matrices Q, R is not worse, than the nonlinear one despite of the opposite statement in [3]. From the other hand the linear controller (25) is much simpler then the nonlinear controllers (21), (21).
This fact in turn allows usage of the linear controllers (with corresponding optimization) in other similar problems of such plants control.

7. Synthesis of the path tracking algorithm

We return to the system (16), (17), which will be written in the form (22). Suppose, that vector of the feedback gains

$$K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}$$

is obtained as a result of the functional (23) optimization. In this case the equation of the controller that provides the closed-loop system asymptotic stability will have the following form

$$u = k_1 y + k_2 \dot{\hat{y}} + k_3 \varphi + k_4 \dot{\hat{\varphi}}. \tag{26}$$

Likewise to the item 3 we will generalize the algorithm (26) on the case of the tracking of the given (program) changing of the coordinate y (or program signal y_p). Namely, let control law will have the following form:

$$u = k_1(y - y_p) + k_2 \dot{\hat{y}} + k_3 \varphi + k_4 \dot{\hat{\varphi}}. \tag{27}$$

However such simple algorithm can't provide sufficient tracking performance, if y_p is comparatively fast changing function.

Consider the problem of the performance improvement in the case [6], when in the low-pass frequency band the main tracking error appears due to the phase lag. We can write the transfer function between the input (y_p) and output (y) of the system, which is described by equations (22) and (27). This transfer function $H(s)$ ($y = H(s)y_p, s = i\omega$) has the following form

$$H(s) = \frac{gk_1}{s^4 + s^3 k_4 + s^2 k_3 + sgk_2 + gk_1}. \tag{28}$$

If the frequencies are relatively low $\omega << 1$ ($s = i\omega$), the magnitude frequency response of the system with transfer function (28) will be close to metricconverterProductID1. In1. In this frequency band the main error between the output of the tracking system (y) and reference input track (y_p) will be determined
$y_p, y - y_p y_p, y - y_p$by phase lag. This lag ν, which is produced by the system with transfer function (28), for small ω is determined by the following expression

$$\nu = -\frac{k_2}{k_1}\omega. \tag{29}$$

The phase delay (29) can be compensated by including in the stabilization algorithm (27) the magnitude of the program signal Figure 6 $y_p(t + \Delta)$ corresponding to some time lead Δ, i.e.

$$u = k_1(y - y_p(t + \Delta)) + k_2 \dot{\hat{y}} + k_3 \varphi + k_4 \dot{\hat{\varphi}}. \tag{30}$$

Indeed, the transfer function $H_1(s)$ corresponding to (30) has the following form

$$H_1(s) = H(s)e^{\Delta s}.$$

The phase lag of this system in the low-pass frequency band (the analog of (29)) may be written as follows

$$\nu = -\frac{k_2}{k_1}\omega + \Delta\omega. \tag{31}$$

Taken Δ from condition$\nu = 0$, we will find in accordance with (31):

$$\Delta = \frac{k_2}{k_1}. \tag{32}$$

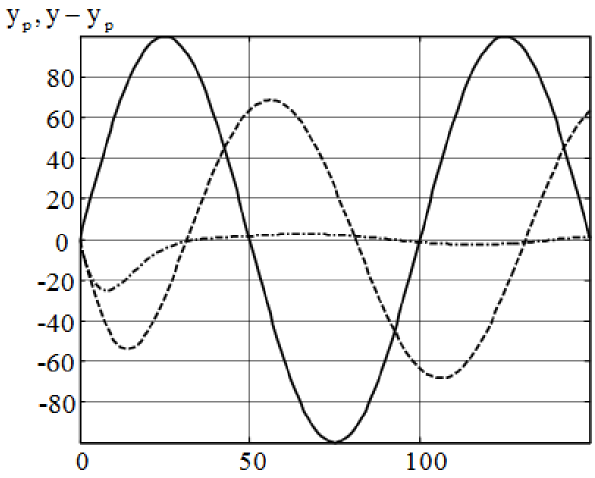

Figure 6.

We will illustrate the efficiency of such approach using example considered in the item 6. We suppose that$y_p = a\sin(\omega t), a = 100, \omega = \frac{2\pi}{100}$. In accordance with (25) we have: $k_1 = 0,01,\ k_2 = 0,1083,\ k_3 = 5,5123,\ k_4 = 6,8574$. Using expression (32), we will obtain $\Delta = \frac{k_2}{k_1} = 10,83$.

The simulation results are given in the Figure metricconverterProductID6. In6. In this Figure the solid line corresponds to the program signal ($y_p(t)$), dash line corresponds to the tracking error ($y - y_p$) when $\Delta = 0$ (control law is determined by expression (27)), and the dash-dotted line corresponds to the tracking error in the case, when control law is determined by (30) and the value Δ is determined by (32), i.e. $\Delta = 10,83$. Results shown in the Figure 6 demonstrate that including in the control law the program signal with the time lead determined by (32) can essentially improve the tracking performance.

Note that in the considered algorithm the restriction on the program signal derivative is absent [12].

8. Control of the quad-rotor speed

We continue the consideration of the quad-rotor motion control along the y-axis. However we will assume that the reference value is not the final point $y_d$on this axis as it was stated in [3], but the reference speed of the quad-rotor motion along this axis $\dot{y}_d$. This problem statement has crucial importance for the design of the UAV path following guidance algorithms [5, 6], because the ultimate goal of the guidance problem solution is the UAV velocity vector control.

Let us formulate corresponding optimization problem. Assume that the quad-rotor motion is described with the equation (22). In order to optimize the transient processes it is necessary to decrease the value of the state variable $\varphi(t)$ and control moment$\bar{\tau}_\varphi(t)$. As far as the $y-$variable is not minimized, it must be excluded from the criterion (23). Taking in account this consideration, the matrix Q in (23) is chosen in the following form:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{33}$$

Proper choice of the weight R and value c in (33) allows the restriction of the values $\varphi(t)$ and $\bar{\tau}_\varphi(t)$ mentioned above.

It is necessary to note that such choice of the matrix Q will lead to the singularity of the Hamiltonian matrix of the variation problem, defined by relations (22), (23). This fact can create some certain difficulties in the solution of the problem of the controller gains determination. Thus, for instance, under such problem statement it is impossible to use the MATLAB LQR-procedure. That is why it is expedient to use the approach proposed in the item 1.4 of the reference [1].

So let matrix U exists, which reduces matrices A, Q to the following form:

$$\bar{\hat{A}} = UAU' = \begin{bmatrix} A_1 & 0 \\ A_2 & 0 \end{bmatrix}, \bar{\hat{Q}} = UQU' = \begin{bmatrix} Q_1 & 0 \\ 0 & 0 \end{bmatrix}.$$

In our case matrix U might be chosen as follows:

$$U = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Solution of the Riccati equation $SA + A'S - SBR^{-1}B'S + Q = 0$, which corresponds to the variation problem determined by (22), (23), must be sought in the form [1]:

$$S = U' \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} U. \tag{34}$$

Matrix S_1 in (34) satisfies to the following Riccati equation:

$$S_1A_1 + A_1'S_1 - S_1b_1S_1 + Q = 0, \tag{35}$$

$$UBR^{-1}B'U' = \begin{bmatrix} b_1 & b_2 \\ b_2 & b_3 \end{bmatrix}.$$

After finding solution of the equation (35) and using relation (34) it is possible to determine controller gains in the following way:

$$K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix} = R^{-1}B'S. \tag{36}$$

The peculiarity of the matrix U structure leads to the following structure of the matrix S, which is determined by (34):

$$S = \begin{bmatrix} 0 & 0 \\ 0 & \tilde{S} \end{bmatrix}.$$

So the first gain in the controller gain matrix (36) will equal zero and the controller equation in this problem has the following form:

$$\bar{\hat{\tau}}_\varphi = -k_2(\dot{\hat{y}} - \dot{\hat{y}}_d) - k_3\varphi - k_4\dot{\hat{\varphi}}. \tag{37}$$

Note, that it is possible to obtain the similar control algorithm to the problem of the quad-rotor motion control along the x-axis.

Example 8.1. We will illustrate the synthesis of controller (37). It is assumed that weight coefficients are: $R = 10$ in (23), and $c = 10^2$ in (33). For these initial data the following expressions for matrix S and controller gain matrix Kwere obtained as follows:

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2.1027 & 11.4597 & 3.1623 \\ 0 & 11.4597 & 205.1464 & 65.1621 \\ 0 & 3.1623 & 65.1621 & 36.2387 \end{bmatrix},$$

$$K = \begin{bmatrix} 0 & 0.3162 & 6.5162 & 3.6239 \end{bmatrix}.$$

The state propagation matrix $(A - BK)$ of the closed-loop system has the following eigenvalues:

$$Z_{cl} = [\ 0, \quad -0.69, \quad -1.4670 \pm 1.5297i\ ].$$

The transient process in this system (plots of the $\dot{y}(t)$ $\varphi(t)$ variables under zero initial conditions) and given value $\frac{dy_d}{dt} = 1$ m/sec is represented in the Figure 7 (the solid line corresponds to the $\dot{y}(t)$-plot, and the dashed line corresponds to the $10 \cdot \varphi(t)$-plot). As it could be seen from this Figure the maximum value of the $\varphi(t)$ variable (roll angle) is sufficiently small ($\cong 4 \cdot 10^{-2}$rad).

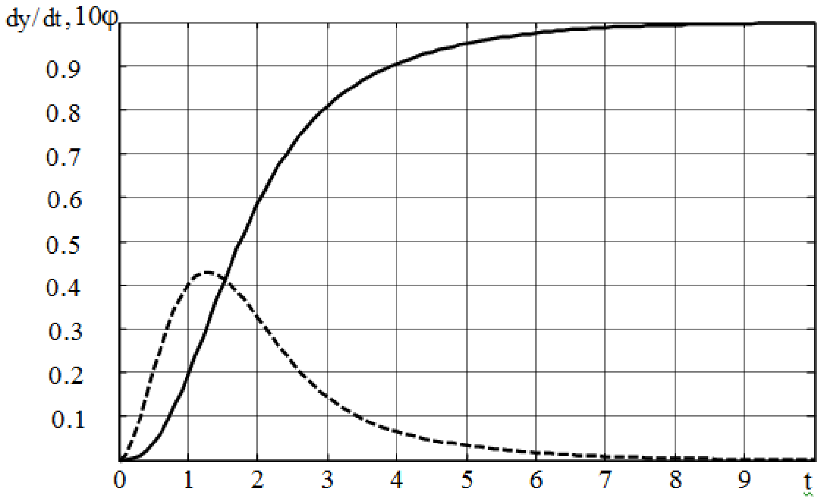

Figure 7.

9. Conclusion

1. The problem of the synthesis of the quad-rotor horizontal motion control is considered for three basic flight control modes: position control, reference path tracking and cruise speed control.
2. In the 1st case it was proposed the control algorithm based on the LQR- synthesis for control of the given final quad-rotor position on the horizontal plane. The comparison of this result with non-linear control algorithm proposed in [3, 9] shows that the proper usage of the LQR-synthesis procedure leads to the quite acceptable results which could be successfully compared with results obtained by the usage of the non-linear algorithm. However that linear control algorithm is simpler than non-linear one and its implementation in the onboard microcontroller is essentially easier.
3. In the 2nd case the algorithm of the reference path tracking by quad-rotor was proposed. This algorithm is based on the results obtained from the 1st case from one hand and on introducing the phase lead in the control law for compensation of the phase lag produced by closed-loop

control system from the other hand. This phase lead essentially improves the path tracking performance.

4. In the 2-nd case the quad-rotor speed control algorithm was synthesized. This control algorithm is very important for the design of the quad-rotor path tracking guidance systems. The peculiarity of this algorithm is the singularity of the Hamiltonian matrix of the corresponding variation problem. The approach for the solution of such kind of problems was proposed in [1]. Using this approach it was possible to find quad-rotor speed controller with optimized transient processes.

References

[1] Aliev, F.A., Larin V.B., (1998), Optimization of Linear Control Systems, Gordon & Breach Sci., Publ., 261 p.

[2] Castillo, P., Lozano, R., Dzul, A., (2004), Real-time Stabilization and Tracking of a Four-rotor Mini Rotorcraft, IEEE Trans. on Control Syst. Techn., 12(4), pp. 510-516.

[3] Castillo, P., Lozano, R., Dzul, A., (2005), Stabilization of a Mini Rotorcraft with Four Rotors. IEEE Control Systems Magazine, pp.45-55.

[4] Gruszka, A., Malisoff, M., Mazenc, F., (2013), Bounded Tracking Controllers and Robustness Analysis for UAVs, IEEE Trans. Automat. Contr., 58(1), pp.180-187.

[5] Khoroshun, A.S., (2016), On Stability of Horizontal Motion of an Airplane, Int. Appl. Mech., 52(1), pp. 134-144.

[6] Larin, V.B., (2006), On Stabilization of Motions of System with Nonholonomic Constraints, J.of Automat. and Inform. Sciences, 38, pp.8-22 .

[7] Larin, V.B., (2016), Navigation of the Wheeled Transport Robot Under Measurement Noise, TWMS J. Pure Appl. Math., 7(l), pp.20-27.

[8] Larin, V.B., Tunik, A.A., (2015), On Improving the Quality of Tracking the Program Trajectory by Aircraft, Int. Appl. Mech., 51(5), pp.137-144.

[9] Teel,A.R., (1992), Global stabilization and restricted tracking for multiple integrators with bounded controls, Systems& Control Lett., 18(3), pp.165-171.

[10] Vos, E., van der Schaft, A.J,. Scherpen, J.M., (2016), A Formation Control and Velocity Tracking for a Group of Nonholonomic Wheeled Robots, IEEE Trans. Automat. Contr., 61(9), pp.2702-2708.

[11] Wahyudie, A., Susilo, T.B., Noura, H., (2013), Robust PID Controller for Quad-rotors. Journ. of Unmanned System Technology, 1, pp.14-19.

[12] Xu, Zhang, Yan, Lin A., (2012), New Approach to Global Asymptotic Tracking for a Class of Low-Triangular Nonlinear Systems via Output Feedback, IEEE Trans. Automat. Contr., 57(12), pp.3192-3196.

Vladimir B. Larin, for a photograph and biography, see TWMS J. Pure Appl. Math., V.2, N.1, 2011, p.160.

Anatoly A. Tunik, for a photograph and biography, see TWMS J. Pure Appl. Math., V.7 N.2, 2016, p.166.